ERC Starting Grant 2023
Research proposal [Part B1]
(Part B1 is evaluated both in Step 1 and Step 2
Part B2 is evaluated in Step 2 only)

Connecting Random Conformal Geometry and Teichmüller theory

RaConTeich

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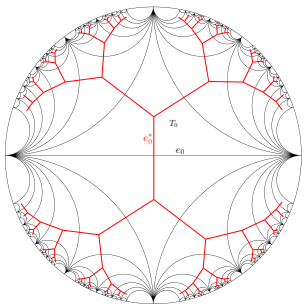
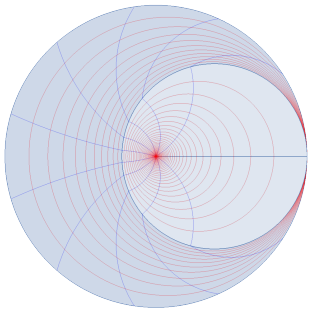
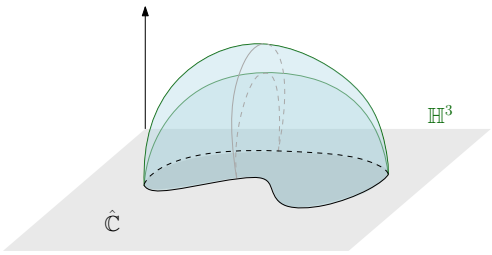
- **Host institution:** Institut des Hautes Études Scientifiques

- **Proposal duration:** 60 months







Random conformal geometry deals with the analysis of conformally invariant systems using probabilistic methods. Random geometric objects, such as the random fractal SLE curves, arise from statistical mechanics models and are of central interest in modern probability theory and mathematical physics. **Teichmüller theory** as originally introduced studies complex structures on a surface. Teichmüller spaces carry a rich geometric structure and have been an important research topic since the mid-twentieth century. These two fields are traditionally far apart. However, via the introduction of the **Loewner energy**, the PI showed surprisingly that SLE is closely related to Weil-Petersson Teichmüller space and made the first contact between SLE theory and Teichmüller theory. Further developments exploring the link and recent progress in probabilistic Liouville conformal field theory also suggest that the connection between the two areas is stronger than currently known and we believe exploring the connection is of major scientific interest.

The ambition of this project is to break new ground in establishing links between fundamental concepts in random conformal geometry and Teichmüller theory by combining in a pioneering way techniques from probability, complex analysis, geometric analysis, Kähler geometry, spectral theory, and representation theory. We focus on three objectives:

1. Advance the understanding of the deep reason behind the link between SLE and the Kähler geometry of Weil-Petersson universal Teichmüller space.
2. Establish the link between Liouville actions and projective structures in a systematic way.
3. Identify the holography of Loewner energy and explore the uncharted territory of holographic principles of random conformal geometry.

The successful completion of this program will substantially reshape our understanding of both areas and has the potential to bring revolutionary development to them both.