

MIT 18.04. COMPLEX VARIABLES WITH APPLICATIONS
SPRING 2021, MWF 2:00-3:00 IN 2-190.

Tentative schedule. Psets are due Tuesday 6 pm.

- (1) **Feb. 17 Topic 0, 1–1.6.1** *Synchronous on Zoom*
Complex number, complex algebra, complex conjugate, vectorial representation, polar coordinate, exponential function
- (2) **Feb. 19 Topic 1.6.2–1.14** *Synchronous on Zoom*
Complex exponential and polar form, complexification, n -th roots, inverse Euler formula, de Moivre's formula, complex function as mapping, argument function, complex logarithm, complex power.
- (3) **Feb. 22 Topic 1.9, 1.11, 2.1–2.3** *Synchronous on Zoom*
Matrix representation of complex numbers, complex functions as mappings, complex derivative, analytic functions.

Pset 1 Due Feb. 23

- (4) **Feb. 24 Topic 2.4–2.7** *Synchronous on Zoom*
Limit, continuity, Riemann sphere, Complex derivative, Cauchy-Riemann equation
- (5) **Feb. 26 Topic 2.8–2.10** *Synchronous on Zoom*
Cauchy-Riemann equation, examples of analytic function and proofs.
- (6) **Mar. 01 Topic 2 end, Review of 18.02**
Domain of definition of analytic function. Gradient, Curl, Divergence, fundamental theorem of gradient field, Green's theorem.

Pset 2 Due Mar. 02

- (7) **Mar. 03 Topic 3**
Complex line integral, fundamental theorem of complex line integral
- (8) **Mar. 05 Topic 3**
Cauchy's theorem
- (9) **Mar. 09 Topic 4** *Monday schedule*
Cauchy's integral formula

Pset 3 Due Mar. 09

- (10) **Mar. 10 Topic 4**
Cauchy's integral formula, application, triangle inequality for line integrals, Cauchy's inequality, Liouville's theorem.
- (11) **Mar. 12 Topic 4**
Applications of Cauchy's integral formula: Fundamental theorem of algebra, circle average of analytic function, maximum modulus principle, maximum principle for real and imaginary part of an analytic function.

(12) Mar. 15 Topic 5

Harmonic function definition, real and imaginary parts of an analytic function and harmonic functions.

Pset 4 Due Mar. 16**(13) Mar. 17 Topic 5**

Maximum principle and mean value property of harmonic functions. Level lines of harmonic conjugates.

(14) Mar. 19 Topic 6 *Add date*

Hydrodynamics. Interpretation of divergence and curl, stationary, incompressible and irrotational flows.

Mar. 22 Student holiday, no class**(15) Mar. 24 Topic 6** Complex potential of incompressible and irrotational flows on a simply connected domain. Examples.**(16) Mar. 26 Topic 7.1-7.4**

Geometric series, power series, radius of convergence of power series. Comparison, ratio and root tests.

(17) Mar. 29 Topic 7.5

Taylor's theorem and consequences

Pset 5 Due Mar. 30**(18) Mar. 31 Topic 7.6-7.8**

Proof of Taylor theorem, isolated singularities, existence of Laurent series.

(19) Apr. 02 Review session *Synchronous on Zoom***Apr. 05 Midterm no class****(20) Apr. 07 Topic 7.8-end**

Laurent series examples, poles, residues.

(21) Apr. 09 Discuss Midterm, Topic 8**(22) Apr. 12 Topic 8**

Cauchy's residue theorem, examples, residue at ∞ , Midterm Q2.

(23) Apr. 14 Topic 9 Two theorems for vanishing line integrals.**(24) Apr. 16 Topic 9** Trigonometric integral, branch cuts.**Pset 6** Due Apr. 16**Apr. 19 Patriots' Day, no class****(25) Apr. 21 Topic 9**

Branch cut, principal value, integral around circular arcs around simple poles.

(26) Apr. 23 Topic 9

Fourier transform.

(27) Apr. 26 Topic 10

Argument principle, winding number.

Pset 7 Due Apr. 27

(28) **Apr. 28 Topic 10**

Rouché's theorem and applications, Counting roots using Rouché's theorem, proof of fundamental theorem of algebra,

Drop date Apr. 29

(29) **Apr. 30 Topic 11**

Conformal mappings, local injectivity, conformal mappings and biholomorphic function

(30) **May 03 Topic 11**

Conformal mappings, fractional linear transformation, mapping circles to circles

(31) **May 05 Topic 11**

fractional linear transformation, reflection with respect to circles, mapping two disjoint circles to concentric circles by FLT

Pset 8 Due May 06

May 07 Student holiday, no class

(32) **May 10 Review session**

Practice exam question 8, Pset 8.4, mapping three points to three points by FLT, harmonic function is preserved under precomposition by analytic function

May 12 Take home exam, no class

(33) **May 14**

Analytic continuation, Riemann zeta function, Euler's product formula.

(34) **May 17**

Riemann surfaces, holomorphic function between Riemann surfaces, Riemann sphere, meromorphic function as holomorphic function to the Riemann sphere.

(35) **May 19**